

An Experimental Study of Free Convection Mass Transfer from High Vertical Surfaces to Liquids

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ABSTRACT

Conventional mining of potash in Saskatchewan required construction of two 10 million dollar shafts to reach ore 3,300 feet below the surface of the prairies. Solution mining of salts does not require such shafts. Water is pumped down to the salt bed through bore holes, dissolves the salt, and then is forced back to the surface where the salt is recovered from the brine.

Solution mining of potash is hampered by the lack of data and theory concerning the dissolution of soluble salts under conditions of free convection mass transfer which exist in underground caverns. Accurate estimates of the dissolution rate of potash ore are required for different temperatures and concentrations under conditions of laminar and turbulent flow.

Squire and Eckert have used the von Karman integral method to solve various mathematical models representing free convection mass transfer at a vertical solid-liquid interface. The method requires a knowledge of the velocity and concentration gradients near the interface. These data were not available so they used corresponding gradients determined in the field of free convection heat transfer.

The Experimental studies reported in this paper were made to check and modify the solutions of the mathematical models. Surfaces of invert sugar as high as 20 feet were used to determine velocity and concentration gradients, dissolution rates, and the location of the transition from laminar flow to turbulent flow free convection mass transfer.

The experimental results revealed that gradients determined in free convection heat transfer were

not always applicable to the corresponding mass transfer case. Theoretical equations derived during the studies predicted dissolution rates from surfaces to be expected in solution mining of potash which were much higher than rates which would have been predicted before these studies were undertaken. Thus there is a justification to complete the study of free convection mass transfer under conditions of turbulent flow.

INTRODUCTION

Although potash was discovered in Saskatchewan in 1943, potash production did not start until 1959. During these sixteen years the provincial government and its agencies encouraged the potash industry to maintain interest in developing this immense ore body. In particular, the Saskatchewan Research Council suggested that solution mining should be investigated. In 1960, a study was undertaken to provide cost data on the feasibility of such a method.

Because of the lack of information on dissolution rates, the Council undertook a study of the mass transfer phenomena associated with the solution mining of a typical potash ore bed in Saskatchewan. Preliminary estimates of dissolution rates of potash ore under free convection mass transfer conditions were made using available theory and experimental data. The results indicate that cavities of certain possible dimensions might be obtained at high enough extraction rates to warrant commercial investigation. With this established, a laboratory study of potash ore dissolution rates seemed justified and experimental work was done to confirm theories concerning the rate of

dissolution and the shape of the cavities. The results of this work have been published (Ozsahin and Husband, 1965), (Husband and Ozsahin, 1967a, p. 234), (Husband and Ozsahin, 1967b, p. 360) and indicated that several key points required further investigation.

In the past, mass transfer under free convection conditions with laminar flow has been considered for smooth vertical dissolving surfaces. Obviously, turbulent flow conditions must be considered for smooth and rough surfaces. Also, equations to predict dissolution rates have been derived on the assumption that the fluid flow and the mass transfer boundary layers have the same thickness. Recent observations show that the former is many times thicker than the latter. Previous researchers used surfaces not more than 6 inches high in their experimental studies, (Fouad and Ibl, 1960, p. 233). One study indicated that transition from a laminar to a turbulent mechanism in mass transfer occurred at a higher value of the Gr Sc product than that of the corresponding Gr Pr product in heat transfer. However, the exact value of the Gr Sc product was not established.

This paper summarizes an experimental study undertaken to investigate more fully mass transfer under free convection conditions from vertical surfaces as high as 20 feet (Husband, 1969). Mass transfer rates under laminar flow conditions and under turbulent flow conditions were measured and boundary layer concentration and velocity profiles were determined. An invert sugar mixture was used as the dissolving substance for experimental purposes. The instrumentation, techniques and experience acquired in this preliminary study will provide valuable background for the next phase involving crystalline materials such as sodium chloride, potassium chloride, and potash ore.

PAST EXPERIMENTAL WORK

Wagner (1949, p. 1030) measured rates of dissolution by suspending thin plates of sodium chloride in water and varying their height from 2.6 to 7.7 cm. Brenner (1941, p. 28) performed a novel experiment with a cylindrical vertical cathode 10 cm high in which he determined the density distribution by quickly freezing the electrolyte. Wilke *et al.* (1953, p. 663) completed a study on free convection mass transfer which compared rates of dissolution of vertical surfaces of organic acids to rates evaluated by the electrolysis of copper and silver solutions at vertical electrodes.

Ibl (1958a, p. 174) studied extensively free convection mass transfer at vertical electrodes. In 1958

Ibl and Muller (1958b, p. 346) published the results of an optical study on the velocity distribution at vertical electrodes 14 cm high. Fouad and Ibl (1960, p. 233) reported the results of a study on free convection mass transfer at vertical electrodes as high as 96 cm. An optical method was used to observe the fluid flow patterns and to detect the onset of fluid turbulence. They found that the diffusion boundary layer did not become hydrodynamically turbulent until a Gr Sc value of 4×10^{11} was reached. In heat transfer, transition from laminar to turbulent flow conditions occurs between Gr Pr values of 10^8 and 10^{10} according to Eckert and Jackson (1951). Fouad and Ibl also found that the mass transfer rate was not affected significantly by turbulent flow conditions up to a Gr Sc value of 4×10^{13} .

Durie and Jessen (1964, p. 275) (1966, p. 326) reported a study on free convection mass transfer from vertical surfaces from sodium chloride as high as 20 cm. The surfaces became rough as dissolution proceeded and measured mass transfer rates increased with time until surface roughness reached an equilibrium value. By extrapolating measured rates back to the instant dissolution started, Durie and Jessen obtained mass transfer rates which agreed well with rates calculated from theoretical equations developed in the same manner as Wilke *et al.* (1953, p. 663). Several experiments using forced turbulent flow down the sodium chloride surfaces produced mass transfer rates several times greater than those obtained under laminar free convection conditions.

EXPERIMENTAL PROCEDURE

Dissolution rates.

Initially slabs of NaCl or homogenized potash ore were prepared from blocks compressed by a local salt block manufacturer. Each slab was approximately 8" x 8" x 1" thick. Dissolution rates were measured initially by gluing a metal hook into the center of one edge, suspending the slab vertically in a tank of water at 25°C, and recording the apparent weight at regular intervals.

Bubbles of air emerged from the dissolving surface during experiments and so some slabs were soaked in saturated NaCl solution overnight prior to being dissolved. Although the pre-soaking reduced the quantity of air emerging as bubbles, it did not eliminate the problem.

Surfaces as high as 34 inches were prepared by fitting 2, 3, 4 or 5 of the slabs together and gluing them to a wooden support. Dissolution tests

showed that the horizontal joints initiated localized intense turbulence in a downward flowing boundary layer. Large holes or cracks appeared along the joints long before the dissolution test was completed. Considerable effort was directed toward the development of a near perfect joint that would eliminate the initiation of turbulence. Although the joints could be improved with careful sanding, near perfect joints were very difficult to fabricate.

The slabs cut from compressed blocks of salt were not considered suitable for obtaining experimental data to check the various theoretical equations for predicting mass transfer rates from high dissolving surfaces. A material was needed which could be cast or fabricated into continuous slabs with smooth surfaces as long as 20 feet. Previous investigators working with small surfaces had used NaCl or slightly soluble organic acids such as benzoic or salicylic acid. These organic acids were not suitable for the present studies as they were expensive and had solubilities less than a tenth of those of NaCl or KCl.

After considerable testing, a satisfactory surface for large scale dissolution studies was found. Slabs of invert sugar up to 20 feet in length were prepared by heating sucrose, water and vinegar in the proportions of 68.8: 2.02: 1 by weight, and boiling the resulting solution until the boiling point reached 145°C. The resulting invert sugar was poured immediately into moulds. Moulds with moveable transverse partitions, as shown in Figure 1, were used and a batch of invert sugar was used to pour a 1 foot long slab in one mould and a 19 foot long slab in the other, or a 2 foot long slab in one mould and an 18 foot long slab in the other, etc. In this manner a constant composition of the invert sugar was insured for all experiments.

Dissolution rates were measured by suspending the invert sugar surfaces in a 24 foot tank which was filled with water at 25°C as the solvent. A piping arrangement was used to fill the tank with water and to bring the tank contents to a uniform temperature (Fig. 2). The final temperature invariably was in a range of 25.0 ± 0.2°C.

The 20 foot slab-mould assemblies were clamped to a 20 foot aluminum I-beam support. The sup-

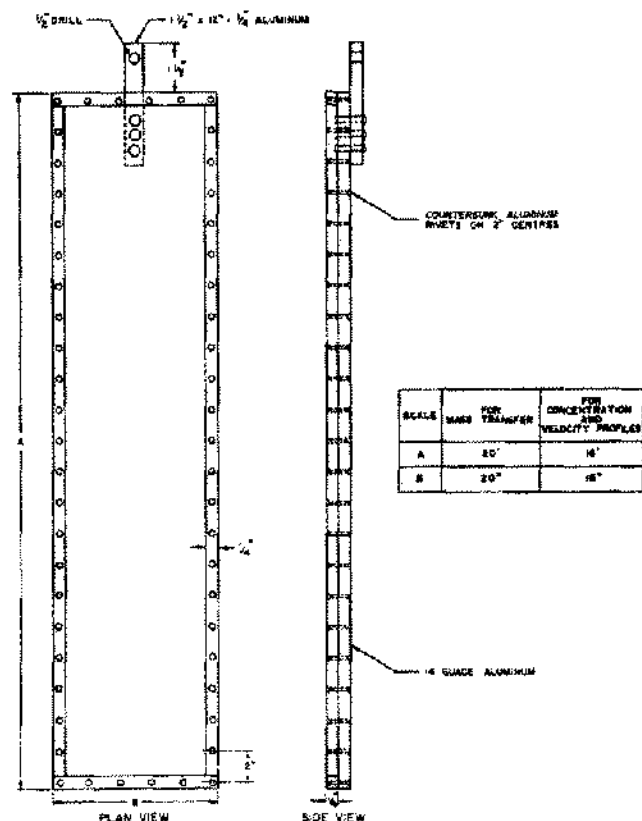


Figure 1. Moulds for invert sugar slabs.

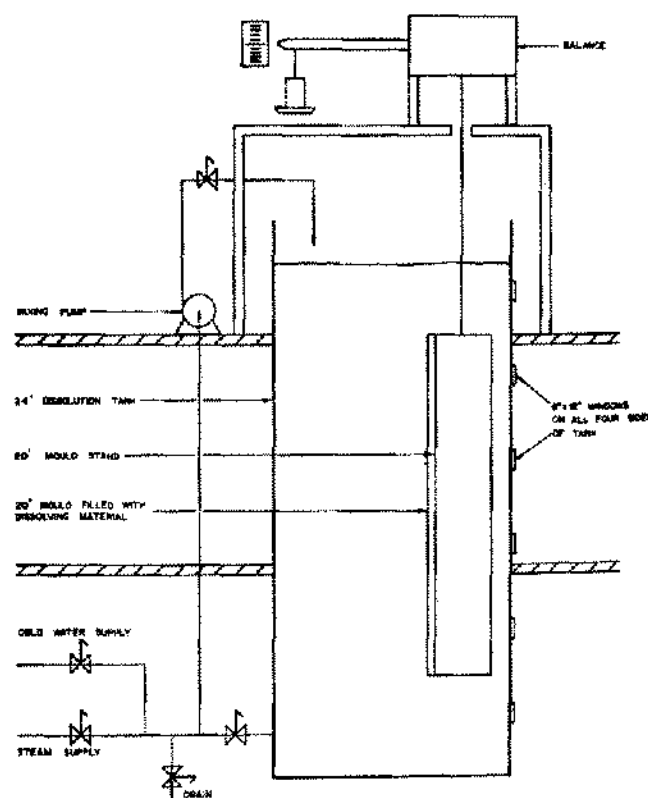


Figure 2. 24 foot tank for mass transfer measurements.

port was hoisted over the tank and then the entire assembly was lowered smoothly into the tank and transferred from the hoist to a suspension mechanism under the balance. The slab-mould assembly was clamped to the 20 foot support to ensure that the invert sugar surface remained perfectly straight and vertical. Apparent weights of the slab-mould-stand combination were recorded at intervals over a 30 minute period.

Concentration profiles.

Concentration profiles have been measured previously by schlieren, interferometric, freezing, and conductivity techniques (Takamatsu *et al.*, 1964, p. 451) (Wranglen, 1958, p. 1543) (Lin, *et al.*, 1953, p. 640). The most successful method for small surfaces is the interferometric method. However, the equipment is expensive and surfaces only 2 or 3 cm wide can be studied.

Capillary type sampling tubes 0.008 inch O.D. and 0.006 inch I.D. were selected. Eleven of these capillary tubes were soldered together in a plane perpendicular to that of the dissolving surface and were moved either toward or away from the dissolving surface by the holder (Fig. 3). This holder was controlled remotely with the use of small electric motors, and a thin feeler probe activated by a vibrator was used to position the holder-probe assembly with respect to the dissolving surface.

Velocity profiles.

Velocity profiles in boundary layers have been measured previously by hot wire anemometers, pitot tubes, and photography of added particles, bubbles or dyes (Fouad and Ibl, 1960, p. 233) (Middlebrook and Piret, 1950, p. 1511) (Popovich

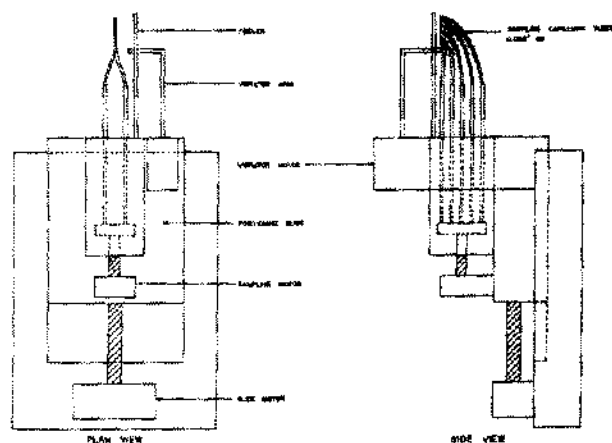


Figure 3. Holder for concentration probes.

and Hummel, 1967, p. 854) (Davis and Fox, 1967, p. 771). Hot wire anemometry is expensive but is especially suited to directional measurements in turbulent flow. Pitot tubes are probes of significant size and in a boundary layer could interfere with the flow pattern. Photography of bubbles, particles or dyes requires magnification of the boundary area under measurement.

A simple variation of the hot wire anemometer was considered in this study whereby a 0.007 inch diameter thermistor bead supported on 0.001 inch diameter lead wires was used. Heat transfer calculations showed that such a sensor would be able to detect low flows when operated at constant temperature, and a simple control circuit could be designed to provide the output signal. By stretching the leads of the thermistor bead between two thin support rods, (Fig. 4) the bead could be positioned in the boundary layer in the same fashion as the concentration probes. As the velocity probes would be used in an aqueous solution, the leads of the thermistor and the support rods had to be insulated so that electrical currents could not enter the solution. The 0.001 inch diameter thermistor

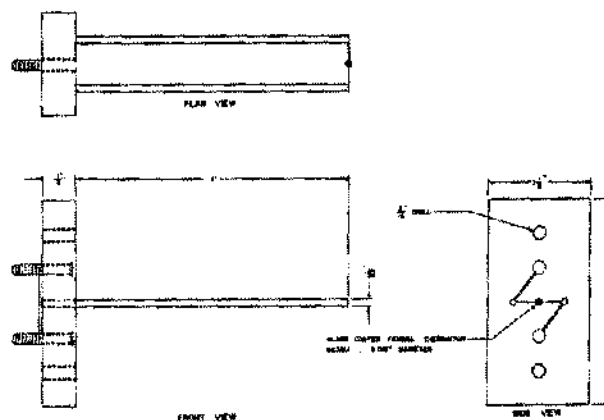


Figure 4. Thermistor velocity probe.

leads proved to be exceedingly difficult to insulate. Finally, the thermistor manufacturer provided units with sputtered glass coatings which were acceptable although the diameter of the thermistor bead was increased to 0.015 inches.

A Wheatstone bridge arrangement was constructed to control the temperature of the thermistor bead (Fig. 5). With the thermistor sensor in fluid at rest at 25°C, $R_{T|V=0}$ and R_V were ad-

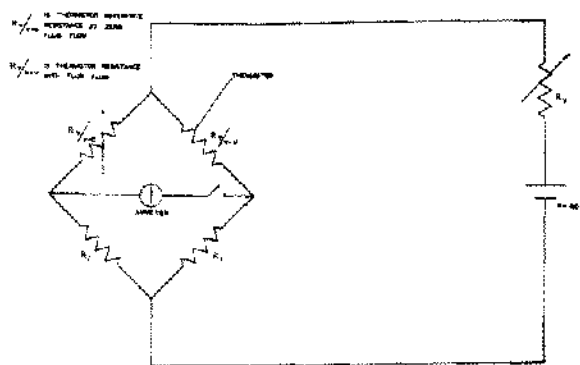


Figure 5. Bridge circuit for thermistor type anemometer.

justed to give a low current through $R_T|_V$ and a null on the ammeter. $R_T|_{V=0}$ was then locked. The value of R_V corresponded to zero fluid velocity. Then with the sensor in fluid flowing at a known velocity, R_V was adjusted until a null was obtained on the ammeter. This value of R_V was a measure of the fluid velocity. The temperature of the thermistor was the same for both measurements. Additional circuitry was provided so that five thermistors could be used alternately.

RESULTS

Dissolution rates were measured for untreated surfaces of the invert sugar, surfaces brushed with water before immersion, and surfaces smoothed with a hot air blower. The results obtained from the latter type of surface have been summarized in Table 1 for surfaces varying in height from 0.25 feet to 20 feet. The data have been arranged graphically in a plot of the log of the average dissolution rate vs the log of the total surface height. (Fig. 6) The results of the concentration measurements have been plotted as concentration profiles at x values of 5 feet, 10 feet, and 15 feet. (Fig. 7)

The results of the velocity measurements 8 feet from the leading edge have been summarized in Figure 8. The points represent the averages of at least three measurements at each particular y . Visual observations were made to obtain an approximate location of the outer edge of the velocity layer. The results are shown in Table 2.

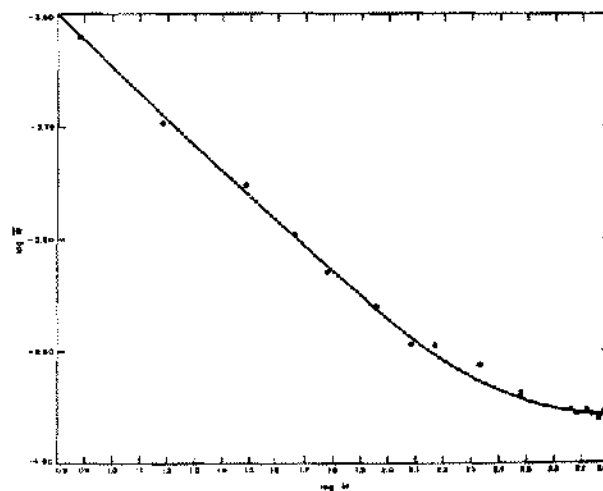
DISCUSSION

Accuracy.

Considerable care was taken to develop a method of accurately measuring the dissolution

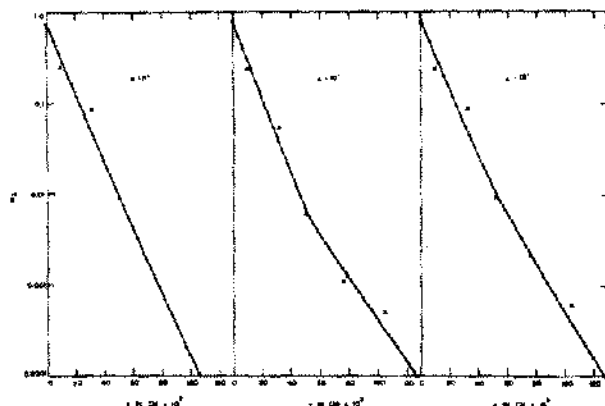
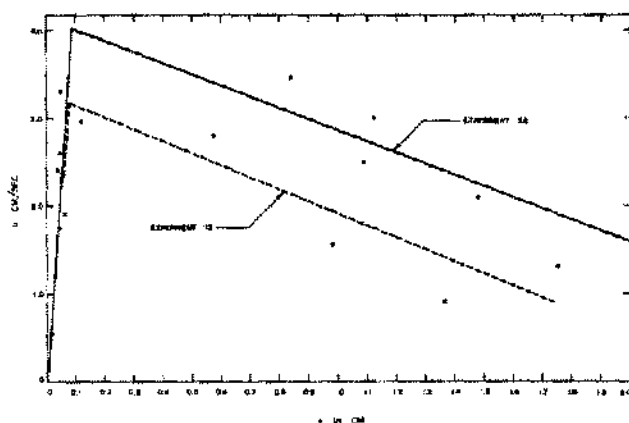
Table 1. Measured Dissolution Rates

| Surface height, H ft | Average dissolution rate at 25°C, m $\frac{\text{grams}}{\text{cm}^2 \text{ sec}} \times 10^4$ |
|-------------------------|---|
| 0.25 | 2.400 |
| 0.5 | 2.017 |
| 1.0 | 1.775 |
| 1.5 | 1.607 |
| 2.0 | 1.484 |
| 3.0 | 1.381 |
| 4.0 | 1.279 |
| 5.0 | 1.270 |
| 7.0 | 1.244 |
| 10.0 | 1.159 |
| 15.0 | 1.118 |
| 16.0 | 1.108 |
| 17.0 | 1.114 |
| 18.0 | 1.111 |
| 19.0 | 1.095 |
| 20.0 | 1.109 |

Figure 6. Log \bar{m} vs log H.

rate for various surface heights. The experimental results for the surface smoothed with hot air show that efforts were successful. m values were reproducible within 5% or less which was more than adequate.

The concentration measurements made at x values of 5 feet, 10 feet, and 15 feet were rea-

Figure 7. w_A vs y .Figure 8. u vs y at $H = 8$ feet.

sonably accurate. Accuracy decreased for y greater than 50×10^{-3} cm because of the low concentrations being measured and the effect of increasing turbulence levels. Precise positioning of the sampling probes, and hence an accurate measure of y , was obtained using the compact probe assembly. Positioning trials showed that the vibrating feeler technique resulted in an error of not more than 2×10^{-3} cm in the value of y .

The results of the velocity measurements with the spherical type of thermistor sensor were disappointing. The initial measurements were made for exploratory purposes and before the sensors had been calibrated for solution other than water. The tests showed that even the concentration boundary layer was turbulent and the turbulence, although of low frequency, had relatively high amplitudes. Manual detection of the null of the

Table 2. Observed Thickness of Velocity Boundary Layer

| Distance from leading edge ft | Boundary layer thickness cm |
|----------------------------------|--------------------------------|
| 2.25 | 6.5 |
| 4.0 | 7.0 |
| 4.5 | 7.5 |
| 6.75 | 8.5 |
| 8.0 | 10.0 |
| 9.0 | 9.0 |
| 12.0 | 11.5 |
| 16.0 | 14.0 |

anemometer of the Wheatstone bridge was difficult because of the turbulent fluctuations and hence not precise.

Concentration profiles.

The only thorough investigation of concentration profiles for free convection mass transfer was made by Ibl (1956, p. 175) using a Jamin interferometric method. Ibl and Müller (1958, p. 346) used these results with the von Kármán method of solving boundary layer equations to produce the following equation for the concentration profile,

$$(C - C_\infty) = (C_0 - C_\infty) \left(1 - \frac{y}{2.1\tau}\right)^{2.3}$$

The general form of this equation was selected arbitrarily to satisfy the von Kármán method and the constants were obtained from experimental studies made with electrodes and dilute solutions of electrolytes.

The results of the concentration measurements in the concentration boundary layer adjacent to the invert sugar surface was correlated in the following equation,

$$(W_A - W_{A\infty}) = (W_{A0} - W_{A\infty}) e^{-8y/\delta_c}$$

This equation differs from that of Ibl and Muller probably because of the high solubility of invert sugar in the concentration boundary layer and the resultant high viscosity. A saturated invert sugar solution contained 5.88 gram moles per litre and had a viscosity of 37.8 poise at 25°C . The electrolyte solutions of Ibl and Muller contained a maximum of 0.5 gram moles per litre and had viscosities around 1 centipoise.

Velocity profile.

Ibl and Müller (1958, p. 346) published the results of an optical study of the velocity distribution near vertical electrodes 14 cm high. This has been the only significant study of velocity profiles for free convection mass transfer reported to date. The experimental results were combined with the von Kármán method of solving the boundary layer equations to produce the following equations for the velocity distribution through the concentration and velocity boundary layers,

$$u = U_1 \left[1.7 \frac{y}{\tau} - \left(\frac{y}{\tau} \right)^{1.7} \right], \text{ for } 0 \leq y \leq \tau,$$

$$u = 0.7 U_1 \left(1 - \frac{y - \tau}{10\tau} \right), \text{ for } \tau \leq y \leq 11\tau.$$

The experimental data of the present study have been arranged into two equations also,

$$u = U_1 \frac{y}{\delta_c}, \text{ for } 0 \leq y \leq \delta_c$$

$$u = U_1 \left(1 - \frac{\delta_c - y}{\delta_c - \delta_v} \right), \text{ for } \delta_c \leq y \leq \delta_v.$$

A comparison of these two sets shows fair agreement within the concentration boundary layer (Fig. 9), but is very poor beyond this layer because

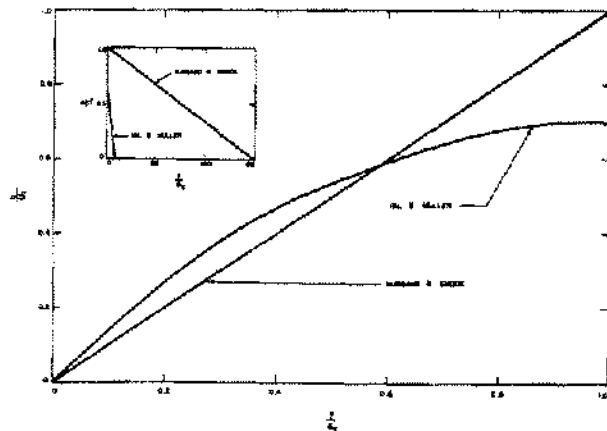


Figure 9. Velocity profiles.

Ibl and Müller have fitted their curves to data from dilute electrolyte solutions.

Dissolution rates.

The data of the dissolution rate measurements have been arranged in the form commonly used to correlate heat and mass transfer rates. A comparison of the correlation of the invert sugar data to those of Fouad and Ibl (1960, p. 233) and Wilke *et al.*, (1953, p. 663) is shown in Figure 10. The plot of the experimental data as Sh number versus the

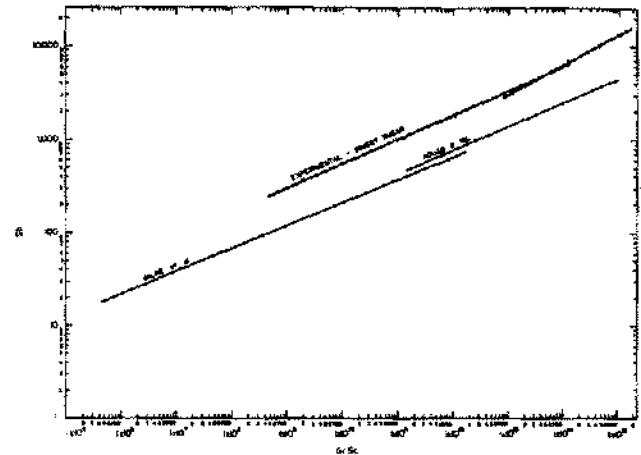


Figure 10. Correlation of free convection mass transfer data.

produce of the Gr and Sc numbers in this figure shows the two regions where mass transfer is affected by laminar or turbulent flow conditions. In the laminar region the data followed the correlation

$$Sh = 1.21 (Gr Sc)^{0.26}$$

and in the turbulent region the correlation is

$$Sh = 0.31 (Gr Sc)^{0.31}$$

The exponents of the Gr Sc group for laminar and turbulent conditions agree well with the experimentally determined values reported in the literature for both heat and mass transfer. The coefficient of the experimentally determined equation for turbulent flow conditions agrees with the corresponding coefficient of Fouad and Ibl for mass transfer but not with the corresponding coefficient of Jakob (1919, p. 530) for heat transfer. The coefficient of the experimental correlation for laminar flow is larger than any of the coefficients determined in other experimental studies. Assum-

ing that the experimental data are correct in all cases, this would mean that the general correlation should include additional functional groups, and variable property boundary layers are involved. The former was indicated by Ostrach (1953) in his rigorous theoretical solution which pertained to constant property boundary layers undergoing free convection heat transfer. The latter was indicated by experimentally determined concentrations, viscosities, and densities which clearly showed that variable property boundary layers existed under the conditions of the invert sugar experiments.

Transition.

To locate the point of transition between laminar and turbulent mass transfer, the local dissolution rate was calculated (Fig. 6). Instead of graphically differentiating the average rate curve, a difference technique equivalent to numerical differentiation was used to determine the local rate of dissolution as a function of x . For an incremental length Δx at a distance x from the leading edge, the local rate of dissolution was calculated from the expression

$$\bar{m}_x = \frac{\bar{m}_{x+\Delta x/2} \cdot (x + \Delta x/2) - \bar{m}_{x-\Delta x/2} \cdot (x - \Delta x/2)}{\Delta x}$$

where $\bar{m}_{x+\Delta x/2} \cdot (x + \Delta x/2)$ is the rate of mass loss for a surface of height $x + \Delta x/2$.

The results of this technique are shown in Figure 11 where Δx was taken as 10 cm. The transition point corresponds to the minimum point of the curve, i.e., where $x = 230$ cm. The determination

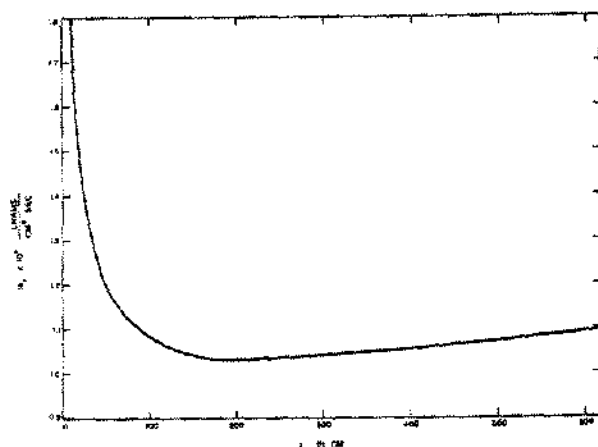


Figure 11. \bar{m}_x vs x .

of the zone of transition between laminar flow and turbulent flow in free convection mass transfer has not been precise in the past. One of the reasons is that turbulence is present to some extent in the boundary layer long before the boundary layer as a whole becomes turbulent. With convective flow over a smooth surface, turbulence will develop initially in the outer regions of the boundary layer and will extend into the inner regions more and more as the flow proceeds upward in the case of heat transfer, or downward in the case of mass transfer. Turbulence must be well developed throughout the boundary layer before it significantly affects the heat or mass transfer rate. Saunders (1936, p. 55) concluded from studies involving air and water that the heat transfer mechanism changed from a form governed by laminar flow to a form governed by turbulent flow at a Gr Pr value of 2×10^9 . Later, Eckert and Jackson (1951) were not so sure when using air, and concluded that the transition could occur between Gr Pr limits of 10^8 to 10^{10} .

In mass transfer Wranglen (1958, p. 536) and Müller (1956) first observed the onset of turbulence at Gr Sc = 4×10^{11} . Fouad and Ibl (1960, p. 233) tried to detect not only the point of initial turbulence but also the point at which the mass transfer changed from a laminar form to a turbulent form. They confirmed that the onset of turbulence for free convection mass transfer occurred at Gr Sc values greater than 10^{11} but they did not clearly establish a mass transfer transition point although their experimental conditions included Gr Sc values as high as 10^{15} . A value of 4×10^{13} was selected as a tentative transition point value for mass transfer but the basis of selection was not given.

No attempt was made in the present study to detect the onset of turbulence in the boundary layer flow. But the point of transition from mass transfer governed by laminar flow to mass transfer governed by turbulent flow has been accurately determined to correspond to a Gr Sc value of 6×10^{13} . The high surfaces used in the present study contributed greatly to the success in locating the transition point accurately, as the Gr number requires H to be raised to the 3rd power in its calculation.

Comparison to theory.

The results of the experimental mass transfer rate measurements on invert sugar surfaces smoothed by hot air treatment were used to compare the mass transfer rates calculated by the theo-

Table 3. Comparison of Theoretical and Experimental
Dissolution Rates of Invert Sugar.

| Surface height, H cm | Average dissolution rate at 25°C, \bar{m} grams cm ² sec $\times 10^4$ | |
|-------------------------|---|------------------------------|
| | Calculated by method of Wilke et al. | Experimentally determined |
| 1 | 4.0 | 3.80 |
| 10 | 2.25 | 2.25 |
| 152.5 (5 ft) | 1.14 | 1.25 |
| 230 (transition point) | 1.025 | 1.195 |
| 244 (8 ft) | 1.01 | 1.18 |
| 305 (10 ft) | 0.95 | 1.15 |
| 457 (15 ft) | 0.87 | 1.12 |
| 610 (20 ft) | 0.81 | 1.109 |

retical equations available in the literature. m values for different surface heights are summarized in Table 3 using the velocity and concentration profiles assumed to date in the literature. To show the general trend with height, \bar{m} values for all heights considered were calculated regardless of whether or not the method applied to laminar or turbulent conditions.

Modifications to the theoretical equations, which include velocity and concentration profiles of the shape determined by this study, will be presented in another paper.

CONCLUSIONS

The majority of the experimental data was sufficiently accurate for the study. The general shapes of the velocity and concentration profiles are satisfactory. The transition between mass transfer controlled by laminar flow and by turbulent flow occurs at a $Gr Sc$ value of 6×10^{13} . Both velocity and concentration profiles should be determined more accurately at values of y or less than 0.01 cm but there is no satisfactory method of doing this at present.

The simplicity of the integral method of von Kármán for solving the boundary equations justifies efforts to improve it. Equations which account for variations in fluid properties such as μ , ρ , and D are needed for the velocity and concentration profiles to improve the method. Examination of ex-

perimental temperature profiles in heat transfer and experimental concentration profiles in mass transfer clearly shows the need for such modifications. A linear profile may be an acceptable approximation for small ΔT and ΔC , but as ΔT and ΔC increase, the curvature of the profiles must become more pronounced.

Correlation of heat and mass transfer rates in the form

$$Nu = A (Gr Pr)^B$$

or

$$Sh = A (Gr Sc)^B$$

is satisfactory for small ΔT and ΔC , although Ostrach has shown that this is not completely correct for laminar flow. The present study and recent heat transfer studies (Warner, 1968, p. 397) show that the correlation is not satisfactory for large ΔT or ΔC where the difficulty of variable fluid properties arises. More complicated functions are required which will account for the variation of fluid properties with distance from the surface.

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NOMENCLATURE

| | | |
|----------------------------|--|--|
| C, C_0, C_∞ | concentration of solute at (x,y) , at interface, and in bulk fluid | $\frac{\text{gram moles}}{\text{litre}}$ |
| D | diffusivity | $\frac{\text{cm}^2}{\text{sec}}$ |
| Gr | Grashof number | |
| H | height of dissolving surface | cm |
| \dot{m}_x | local mass transfer rate at x | $\frac{\text{grams}}{\text{cm}^2 \text{ sec}}$ |
| \bar{m} | average mass transfer rate for a dissolving surface | $\frac{\text{grams}}{\text{cm}^2 \text{ sec}}$ |
| Nu | Nusselt number | |
| Pr | Prandtl number | |
| R_v | variable resistance | ohms |
| $R_T _{V=0}, R_T _V$ | thermistor resistance at temperature T with no flow and with flow of V cm per sec | ohms |
| Sc | Schmidt number | |
| Sh | Sherwood number | |
| T | temperature | $^{\circ}\text{C}$ |
| u | velocity in x direction | cm/sec |
| U_1 | characteristic velocity in x direction | cm/sec |
| $w_A, w_{A0}, w_{A\infty}$ | mass fraction of solute A at (x,y) , at interface, and in bulk fluid | |
| x, y | Cartesian co-ordinates, x parallel and y perpendicular to dissolving surface with origin at leading edge | |
| δ_c, δ_v | thickness of concentration and velocity boundary layers | cm |
| μ | viscosity | $\frac{\text{gram}}{\text{cm sec}}$ |
| ρ | mass density of solution | $\frac{\text{gram}}{\text{cm}^3}$ |
| τ | y corresponding to maximum value of u | cm |